Multiobjective MILP Model for Optimal Allocation of Automated Switching Devices in Electric Power Distribution Systems

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Abstract—This paper presents an analytical mixed-integer linear programming model to improve reliability of distribution networks through the optimal allocation of automated switching devices. Optimal reliability is formulated as a multi-objective problem of minimizing customers' interruptions and the investment costs related to devices' acquisition, relocation and operation. In order to establish the optimal trade-off between these conflicting objectives the Goal Programming approach is proposed. The presented model considers post-fault restoration constraints which limit load transfers and thereby the system reconfiguration capability. Restoration feasibility is ensured by a linear formulation of power flow equations as functions of geographical locations of automated switching devices in the feeder. Therefore, the model's solutions provide a more effective and economic application of automated switching devices, so that voltage and system capacity constraints are taken into account in selecting optimal locations of devices. The proposed methodology was validated using the IEEE 123-bus test feeder.

Index Terms—Allocation of automated switching devices, Distribution systems reliability, Load restoration, Mixed-integer linear programming.

I. INTRODUCTION

The increasing demand for a high-quality energy supply, deregulation of the electric power industry and technological innovation are some factors that have driven changes on traditional paradigms for Electric Power Distribution Systems (EPDS) planning and operation. In this sense, there is a general agreement that EPDS will move from passive and electromechanically-controlled systems to active and intelligent networks in the next two decades [1]. The concept of “smart grids” has been used in reference to networks that incorporate active power sources and automation technologies with integrated functions of monitoring, control and protection.

Among the features of a smart grid is the ability to self-heal, i.e., to carry out fault isolation and self-reconfiguration in a fast and automated manner. Self-healing is a key component to meet reliability requirements of smart grids and is enabled by application of Automated Switching Devices (ASDs) [2]. Currently, ASDs functionalities are available in most modern electronic-controlled line reclosers that incorporate sensing, protection and switching capabilities [3]. These devices have shown to be economically viable due to the emergence of a large number of manufactures and the development of new communication technologies [4].

Designing a flexible distribution network that is able to self-heal is accomplished by a suitable restoration planning [5]. It includes selecting number and locations of ASDs to be installed along the network, so that restoration of out-of-service loads is carried out with a minimum load shedding. Network constraints have important role in the restoration process, since they limit the system’s ability of reconfiguration. Inadequate voltage levels and insufficient tie points’ capacities may prevent reliability benefits from automated load transfers, thus compromising the effectiveness of a self-healing scheme [1]. For the purposes of evaluating the restoration constraints, a power flow analysis is required.

The just described characteristics make allocation of ASDs a combinatorial, constrained and multi-objective problem, whose complexity falls into the class of the NP-Hard problems [6]. Some approaches have been proposed in the current literature, that use heuristics [4], analytical models [7, 8], immune algorithms [9] and reactive tabu search [10] as solution techniques. Some formulations [8-10] address the problem without considering constraints related to restoration. Thus, they cannot ensure that application of ASDs will be effective for load transfer through capacity-constrained tie points. The technique proposed in [4] in turn, has the drawback that it does not ensure the solution optimality, since a heuristic-based approach is considered. None of the previous cited techniques have considered the effect of the protection system in modeling reliability of EPDS. However, it is worth note that for a given fault location, the size of the out-of-

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service area depends on the arrangement of the protective devices along the feeder. Thus, restoration planning is closely related to the protection system design [11]. Other studies have addressed this issue by allocating protective devices [12-14], but in conjunction with manually-operated switches and without considering constraints related to restoration.

This paper proposes a multi-objective model for optimal allocation of EPDS reliability that considers the interrelated effects of the protection system response to faults and the post-fault restoration in determining number and locations of ASDs. Protection and restoration are formulated as Mixed-Integer Linear Programming (MILP) models. The protection model represents the effect of fault clearing by ASDs and has as a result the component of the SAIFI index [15] related to the protection system response to faults. The restoration model provides the subtractive component of SAIFI and represents the impact of the fast load transfer performed by ASDs. Restoration is subject to constraints that ensure that operational limits of the system’s components are not violated with topology reconfiguration. These include tie-points capacities, voltage magnitudes and radial operation. To evaluate such parameters, a linear formulation of power flow equations is developed. The optimal level of reliability is determined as the trade-off between the reliability benefits of ASDs applied for protection and restoration purposes, and the costs of the devices operation, acquisition and relocation. The optimization problem is stated as Goal Programming (GP) model [16] and solutions obtained from a Branch and Bound-based optimization package [17]. The proposed model is validated using the IEEE 123 bus test feeder [18] and solutions evaluated under varying scenarios of system capacity.

II. PROPOSED MODEL FOR OPTIMAL ALLOCATION OF AUTOMATED SWITCHING DEVICES

This section presents the analytical development of the MILP model that represents the effects of ASDs protection and restoration functions on the reliability of EPDS. Thereafter the model of the SAIFI index is aggregated to the ASDs cost model, and stated as a GP model. It should be noted that in contrast to heuristic [4] and meta-heuristic approaches [9, 10, 14], MILP formulations guarantee convergence to the global optimum solution [16]. Also, efficient commercial solvers with large-scale capabilities are currently available which use exact algorithms, such as Branch and Bound to solve the MILP problems [17].

A. SAIFI and ASDs Cost Models

In this paper an analytical simulation technique [11] is used to model the sequence of events that succeed a contingency. By allowing calculation of the impact that the contingency has on each component of the system, the SAIFI index is expressed as a function of basic reliability parameters and binary variables that represent the feeder sections where ASDs are located. Overcurrent protection impacts on reliability by limiting the effect of faults, thus minimizing the number of interrupted customers. In formulating the protection system response it is assumed that proper coordination between ASDs is always attained, so that the device closest to the fault will operate. Consider Fig. 1 (a) which represents a feeder F1. At the substation node (CB1) the main feeder protection is performed by a set of relays. A fault occurrence at node i will cause operation of ASD1 located in section jk. In this case, the number of customers that will experience a sustained interruption is represented by \( N_{ij} \), i.e., the customers downstream from ASD1. The binary decision variables indicating the feeder sections where ASDs are installed are defined according to (1):

\[
s_{jk} = \begin{cases} 
0, & \text{ASD installed in section } jk. \\
1, & \text{otherwise.} 
\end{cases}
\]

Figure 1. Analytical simulation of the protection system response to a faults (a) and restoration (b).

Taking into account the binary variables (1) the impact of fault clearing by the protection system on the SAIFI index is expressed as:

\[
N_p = \frac{1}{N_F} \sum_{i \in G} \lambda_i \sum_{\{j \in U_i\}} N_{ij} s_{jk} \prod_{\{mn\} \in (U_i - U_j)} s_{mn},
\]

where \( \lambda_i \) is the permanent failure rate of node i (failures/year); \( N_F \) is the total number of customers on the feeder; \( G \) is the set of all feeder nodes and \( U_i \) is the set of sections that precede node i (sections upstream from node i) to node 1, which is defined as the feeder root node. The operator \( \{U_i - U_j\} \) denotes the complement of \( U_i \) in \( U_j \), and results in a set of the elements of \( U_i \) that are not elements of \( U_j \).

Equation (2) estimates the contribution to the SAIFI index of the permanent faults in node i when an ASD is installed in the circuit section jk (upstream from node i) and there are not ASDs installed in sections mn (between nodes i and j).

Restoration is the emergency action that follows the fault clearing by a protective device. It consists in a sequence of switching operations that result in a temporary configuration for the network, which is maintained for the time necessary to repair the cause of the interruption. In this paper it is considered that load transfer to an alternative feeder is performed through a “normally opened” ASD, which defines a
The restoration model formulation is described through Fig. 1 (b) that shows the post-fault state of feeder F1. For now it is assumed that restoration of all loads located between ASD1 and ASD2 does not violate any system constraints. In this case, for any faults occurring upstream from ASD1 (between nodes F1 and k), restoration of loads will be performed by opening ASD1 and by closing ASD2. A set of binary variables are defined in order to indicate feasibility of restoration of load point:

\[ r_i = \begin{cases} 1, & \text{feasible restoration of load point } i. \\ 0, & \text{otherwise.} \end{cases} \]  

Equation (3) provides an estimate of the number of customers that are restored along a year, when sustained interruptions are resulting of permanent faults occurring upstream from node j (failures/year); and \( N_i \) is the number of customers at the load point i.

Equation (4) provides an estimate of the number of customers at the load point i:

\[
N_R = \frac{1}{N_i} \sum_{i \in G} N_i r_i \sum_{(jk) \in U_i} \lambda_j^* s_{jk} \prod_{mn \in \langle (U_j, U_j) \rangle} s_{mn}, \tag{4}
\]

where \( \lambda_j^* \) is the sum of permanent failure rates of nodes upstream from node j (failures/year); and \( N_i \) is the number of customers at the load point i.

Economic criteria must be aggregated to limit the utility investment costs to improve reliability. The costs model considers the possible existence of ASDs previously installed on the feeder. Thus, the required cost for improving reliability is the sum of the costs for ASDs operation, relocation and acquisition, expressed as:

\[
\text{COST} = (c_{ac} + c_{op}) \sum_{\{i\} \in G} (1 - s_{ij}) + c_{re} \sum_{\{i\} \in S} s_{ij} - c_{ac} |S| \tag{5}
\]

where \( c_{ac}, c_{op} \) and \( c_{re} \) are the unit costs of ASDs acquisition, operation and relocation, respectively (US$); \( S \) is the set of sections where the existing ASDs are installed; and \( |S| \) is the number of elements of set \( S \).

Figure 2. Line section model (a) and generic node model (b).

**B. Restoration Constraints**

The nonlinear nature of the problem addressed in this paper requires simplifications in the model’s formulation, otherwise optimality cannot be ensured. Thus, a linear approximation of the power flow in the post-fault system is proposed. In part, this assumption is justified by the fact that in emergency situations the operational requirements of the network are not strictly the same as in normal operation. Thus, some constraints such as voltage levels and overloads may be relaxed during the restoration process [5].

The power flow formulation assumes a single-phase representation of the network. Admittances, voltages, currents and powers are expressed in the per unit system (pu), in a common MVA base. Fig. 2 (a) shows a line section connecting nodes i and j, where \( V_i, V_j \) are the nodal voltages, \( I_{ij}, I_{ji} \) are the nodal currents and \( y_{ij} \) is the line admittance. The generalized node model shown in Fig. 2 (b) is used to represent loads and tie points, the later through a Norton equivalent. The node model is characterized by a current injection (I_d) and shunt admittance (y_d). Assuming h as the neighbor node of i (not shown in Fig. 2) the nodal voltage equation for node i is expressed as:

\[
-V_i y_{ih} + V_i \left(y_{ih} + y_{ij} + y_{ji} \right) - V_j y_{ji} - I_{ih} + I_{ji} - I_d = 0. \tag{6}
\]

Alternatively, in matrix form (6) becomes:

\[
Y_{bus} \cdot V_{bus} + I_{bus} = 0, \tag{7}
\]

where \( Y_{bus}, V_{bus} \) and \( I_{bus} \) are the bus admittance matrix, the nodal voltages and the nodal currents vectors, respectively.

The power flow (7) is straightforward, and does not contain information regarding ASDs locations. As shown in Fig 2 (a), allocation of an ASD in a section \( j \) can be modeled as two controlled current sources that obey the relation (8):

\[
I_j = \begin{cases} (V_i - V_j) y_{ij}, & \text{if } s_{ij} = 0, \\ 0, & \text{if } s_{ij} = 1. \end{cases} \tag{8}
\]

Thus, the power flow dependency with ASDs locations is imposed by the following constraints (9)-(11):

\[
I_{ij} \leq (V_i - V_j) y_{ij} + V_{max} y_{ij} s_{ij} \tag{9}
\]

\[
I_{ij} \geq (V_i - V_j) y_{ij} - V_{max} y_{ij} s_{ij} \tag{10}
\]
\[ I_y - I_{ji} = 0. \]  \hspace{1cm} (11)

Where \( V_{\text{max}} \) is the maximum allowable system voltage.

In addition to (9)-(11), constraints (12)-(18) are considered as criteria for restoration feasibility.

- Current capabilities of ASDs:
  \[ I_y \leq I_{\text{max}}^{ij} s_y \]  \hspace{1cm} (12)
  \[ I_y \geq -I_{\text{max}}^{ij} s_y \]  \hspace{1cm} (13)

where \( I_{\text{max}}^{ij} \) is current-carrying capability of the ASD in section \( ij \).

- Allowable voltage range:
  \[ V_i \geq V_{\text{min}} r_i \]  \hspace{1cm} (14)
  \[ V_i \leq V_{\text{max}} r_i \]  \hspace{1cm} (15)

where \( V_{\text{min}} \) is the minimum allowable system voltage.

- Capacity limits of tie points:
  \[ -I_i \leq I_{\text{max}}^i r_i \]  \hspace{1cm} (16)
  \[ -I_i \geq 0, \]  \hspace{1cm} (17)

where \( I_{\text{max}}^i \) is the current capability of a tie-point at node \( i \).

- Radial topology of the post-reconfiguration network:
  \[ \sum_{\{k\} \in P_{kl}} (1-s_y) - r_k - r_l + I \geq 0, \ k \neq l. \]  \hspace{1cm} (18)

Where \( P_{kl} \) is the set of feeder sections in the path between tie-nodes \( k \) and \( l \).

C. Goal Programming Model

Goal Programming is based on the concept of meeting a number of objectives in order to get as close as possible of their goals. Goals are selected so that they cannot be reached simultaneously. The function to be minimized is the weighted sum of the deviations in relation to their respective goals [19]. The proposed GP model is given by (19).

\[
\begin{align*}
\min & \quad w_S \delta_S + w_C \delta_C \\
\text{s.t.} & \quad \text{SAIFI} - \delta_S = g_S \\
& \quad \text{COST} - \delta_C = g_C \\
& \quad \delta_S, \delta_C \geq 0 \\
& \quad s, r \in C_r,
\end{align*}
\]  \hspace{1cm} (19)

where:
- \( w_S \) and \( w_C \) are normalizing factors;
- \( \delta_S \) and \( \delta_C \) are deviations from the of SAIFI and COST functions in relations to the respective goals;
- \( g_S \) and \( g_C \) are the goals of SAIFI and COST functions;
- \( s, r \) represent the problem’s solutions; and
- \( C_r \) is the set of constraints (7), (9)-(18).

The goal \( g_S \) is selected as the minimum possible value assumed by the SAIFI index, which results in:

\[ g_S = \sum_{i \in G} \lambda_i N_i. \]  \hspace{1cm} (20)

In turn, the goal \( g_C \) is selected considering the total cost of operation of the ASDs previously installed in a feeder, as well as the investment that a utility is willing to pay for the reliability improvement. Weighting factors \( w_S \) and \( w_C \) are selected to ensure appropriate tradeoffs between objectives and to perform scaling of the deviational variables. The latter is important to overcome the unintentional bias towards the objectives with a larger magnitude (incommensurability). Weights in (19) are then defined as the upper bounds of SAIFI and COST functions, according to (21) and (22), respectively:

\[
\begin{align*}
w_S &= \left[ \sum_{i \in G} \lambda_i N_i \right]^{-1} \\
w_C &= \left[ c_{\text{op}} \sum_{i \in G} \left(1 - s_y\right) \right]^{-1}
\end{align*}
\]  \hspace{1cm} (21)

III. TESTS AND RESULTS

The proposed model was tested using the IEEE 123 bus system [18] shown in Fig. 3. The permanent failure rates (\( \lambda \)) equal to 0.17 failures/year and the number of 30 customers (\( N \)) were considered for all load points. Voltage levels were considered acceptable within the range from \( V_{\text{min}} = 0.93 \) pu to \( V_{\text{max}} = 1.05 \) pu. Costs of ASDs operation, relocation and acquisition were assumed as equal to \( c_{\text{op}} = \$5,412 \), \( c_{\text{re}} = \$1,250 \) and \( c_{\text{ac}} = \$25,000 \), respectively [13].

Except for the substation node 150, voltage regulators along the feeder were considered bypassed. In these conditions, a power flow analysis was performed in order to evaluate voltage and active power injected by substation node in the system’s normal operating condition. As a result, the voltage of 1.044 pu was assumed for node 150. The proposed model was applied considering scenarios of tie nodes capacities limited to 10% and 25% of the substation node active power injection of 3600 kW. Tie nodes are represented by nodes 195, 251, 350 and 451 in Fig. 3.

For comparison purposes, the six switches previously allocated on the feeder were considered as ASDs. Their original locations as shown in Fig. 3 will be referred to as the base case. Table I presents the results obtained from the
The goal value of the cost objective (gC) includes for each scenario, the tie point capacities (TP Cap.) and the evaluation of the base case and the solutions of the proposed model. It includes for each scenario, the tie point capacities (TP Cap.), the goal value of the cost objective (gC), the SAIFI index value, the percent reduction of SAIFI in relation to the base case (ΔSAIFI), the value of the cost function (COST), the percent difference between the cost function and its goal (ΔCost) and the locations of ASDs. For scenarios 1 and 3 the costs’ goal values were selected as equal to the operation cost of the six ASDs of the base case (US$ 32.472). For scenarios 2 and 4, it was considered the increase of 20% of the goal values, which can be assumed as an additional investment for reliability improvement.

From Table I it can be seen that the solution obtained in scenario 1 resulted in a reduction of 12.07% in the SAIFI in relation to the base case, at the negligible additional cost of US$ 2.500 for relocation of two ASDs. The total cost of US$ 34.972 was increased 7.7% in relation to its goal value. By increasing the goal of the cost function, solution from scenario 2 has determined relocation of the six ASDs previously installed on the feeder, as shown in Fig. 4. In this case, ASDs locations were selected so that restoration is feasible for the regions delimited by the devices. The region formed by the ASD in section 23 encompasses a total load of 282 kW which can be restored through tie node 251. Tie nodes 350 and 451 support loads within the area limited by two and three ASDs, respectively. The injected powers through the tie nodes to restore loads in their respective areas are equal to 278 kW for node 350 and 343 kW for node 451. Node 195 does not contribute for improving the feeder reliability, since it was not possible to obtain a feasible configuration of the network that resulted in an isolated area in which loads could be transferred to the tie point.

From scenario 1 to 3, the improvement on reliability was most due to the increased load transfer capability of the system thus resulting in a reduction of 34.65% of the SAIFI in relation to the base case. The increased capacity led to a better trade-off between the SAIFI and COST functions, as it can be noted by the increased cost of the solution, a consequence of the relocation of three ASDs. Fig. 5 shows locations of ASDs determined by the solutions of the proposed GP model for scenario 4. From Table I it can be seen that the additional investment aggregated to the cost goal does not result in a significant improvement in the SAIFI index. Although the six ASDs were relocated, the total capacity of tie points is sufficient to almost eliminate load transfer constraints. Thus reliability no longer improves, unless a higher amount of investment is applied.

In order to evaluate the errors introduced by the linear approximation of the power flow, the backward-forward sweep technique [20] was applied in each region qualified by the proposed GP model as feasible for restoration. Results indicated that errors introduced in voltages and load currents tend to increase proportionally to the distance from the source node, as well as to the amount of load supported by the source.
Voltages obtained by the proposed linear model presented errors less than 5% in relation to the ones obtained from the non-linear power flow for all scenarios. In turn, errors associated to the calculation of line currents presented a maximum value of 7%.

In the tests cases presented in this paper, solutions were obtained by using the Gurobi Optimizer [17] optimization package, provided on-line through the NEOS-Server for Optimization [20]. For all cases, optimal solutions were found in time intervals of less than 0.2 s, with all control parameters of the algorithm maintained in their default values.

IV. CONCLUSIONS

This paper presented a Goal Programming model for optimal allocation of ASDs in EPDS. Its main features are to account for the effects of protection and restoration capabilities of ASDs on the network reliability, also considering load transfer constraints. The proposed methodology finds its major contribution in formulating the power flow constraints associated with the system reconfiguration as functions of ASDs locations in the distribution feeder. Tests have shown that the model is effective and the formulation of the restoration problem in a goal programming structure is suitable for establishing the trade-offs between reliability and the associated costs. Thus by applying the proposed model utilities can ensure that pertinent expenditures are made in order to achieve the highest level of system’s reliability. Future works should investigate integration of switching times in order to consider the simultaneous allocation of ASDs and manually-operated switches. Also, errors introduced by the linear approximation of power flow should also be investigated.

REFERENCES